



## **Applied techniques of geometric construction in free hand sketches – a course of Technical Drawing for Engineers**

Guilherme W. Lebrão\*  
Gustavo M. Razzante Filho\*  
Jorge Kawamura\*  
Roberto Scalco\*

\* EEM CEUN-IMT - Escola de Engenharia Mauá, Ciclo Básico Disciplina de Desenho - Centro Universitário do Instituto Mauá de Tecnologia, Brazil  
[guinet@maua.br](mailto:guinet@maua.br), [gustavo.razzante@maua.br](mailto:gustavo.razzante@maua.br), [jk@maua.br](mailto:jk@maua.br),  
[roberto.scalco@maua.br](mailto:roberto.scalco@maua.br)

### **RESUMO**

O uso de sistemas digitais de editoração gráfica (Computer Aided Design) popularizou-se nos últimos anos, revolucionando o desenho apoiado em instrumentos como réguas, esquadros e compassos. Os recursos associados à maneira convencional do desenho foram incorporados aos programas de computador, fazendo com que o processo se tornasse mais rápido e mais eficiente. Entretanto, o esboço continua a ser uma ferramenta essencial para a sensibilidade do desenhista no projeto. De fato, é usado ainda como uma ferramenta extremamente rápida e prática na comunicação entre engenheiros, arquitetos e outros designers. Este artigo considera o uso do paralelismo, do perpendicularismo, da tangência e de relações entre os elementos e suas propriedades de geometria plana aplicados na criação dos esboços, feita no plano ou em projeção isométrica. Adicionar estes recursos confere ao esboço uma técnica que incorpore propriedades geométricas, elevando o nível e a precisão da construção. Nesta proposta, utiliza-se uma grade em vez de réguas e demais instrumentos tradicionais, para permitir a construção de quadrados com circunferências inscritas que são a referência básica principal do esboço. Usando arcos da circunferência, é possível construir também ângulos com precisão 15° e aproximações 5°, assim como demais construções geométricas entre arcos de circunferências e segmentos de retas. O uso desta técnica em um curso de desenho para engenharia significa um recurso extraordinário nos esboços à mão livre, melhorando a noção da proporção e visualização dos estudantes, a

construção geométrica básica, o aumento da habilidade de desenho e a representação mais clara.

**Palavras chave: Esboço, Construção Geométrica, Geometria, Desenho Técnico.**

## **ABSTRACT**

The use of Computer Aided Design had become wide popular these last years, leaving the support instruments, such as rulers, square, compasses and protactors. The resources associated to the conventional way of drawing had been incorporated on software, making a faster whole process and more efficient. However, the sketch continues to be an essential tool to improve the designer's sensibility in the project. In fact, it is an extremely fast and practical still used as a communication tool between Engineers, Architects and other designers. This paper considers the use of parallelism, perpendicularity, tangency and concurrent relations between the elements and its deriving properties of Plane Geometry applied in the creation of sketches, made on plan or isometric projection. Adding these resources allows, the development of a technique that incorporates geometric properties to the sketch, raising the drawing level of information. The proposal is to use a grid instead of the rules and all the support instruments to enable the construction of a square inscribed circumferences in, which will happen to be the major reference to drawing. Using these circumference's arcs, it is possible to construct angles with  $15^\circ$  precision and  $5^\circ$  approaches, stand out as well the constructions or concurrent between circumferences arcs and line segments. The use of this technique in an engineer drawing course implies in resources at free hand sketches, improving to designers the proportions and visualization notion, basic drawing construction, geometric ability increase and clear representation.

**Keywords: Sketch, Geometry Techniques, Geometric Construction, Technical Drawing.**

## **1 Introduction**

The Technical Drawing Discipline of Escola de Engenharia Mauá – IMT is taught at first grade, in the basic cycle, for all the engineering qualifications. The discipline's objectives are the development of the sketch abilities and space visualization for the reading, interpretation and confection of engineering projects.

The student works with bidimensional and three-dimensional representations. The course contemplates the cylindrical projections of orthographics representation and isometrics perspective, beyond cuts and sections.

The lessons are given in proper laboratories with 40 students' capacity, provided with computers, for the professor and the students, and projector. The course is 70% of hand-free sketch and 30% computer [3] [4].

The sketch is supported in bidimensional grid in orthographics representation and three-dimensional for the isometrics perspectives representation. The computers are used as electronic plane table, where the student has the first contact with the computer with a CAD system, and as generating of three-dimensional models, a powerful tool for the the space visualization development.

The lessons are practical, exercises basically.

## **2 Discipline Context**

Since 1996 the course always comes if developing privileging the sketch to the hand-free one as a way for the abilities evolution and using the computer as only tool in the aid to the confection of projects; the rulers then had been abandoned.

The students profile in this decade comes moving significantly, and the relative adaptations have been made progressively. The main one of them, in whom it says respect to the drawing specifically, constitutes in supplying a bigger unprepared ness of the students in the knowledge of plain geometry and the manual ability to draw, even when rulers supported. This deficiency became, in part, consequence of the change in the resume of average education for the Laws of Direction Lines and Bases removing the obligator ness of the knowledge Drawing and, therefore, affecting directly disciplines as Geometric Drawing in the Mathematics of the middle cycle [1].

These deficiencies had become patent for the college in 2000, when it was decided to invest more in sketch and geometry.

The initial ideas of a project frequently are performed in simple drawings, made to the hand-free one in the first instant of the creation. Having this sketch ability is a basic skill to the professional of project; therefore it gives to creation rapidity and flexibility. The sketch to the hand-free one is irreplaceable in this process [3].

The load of manual work was increased and from 2001 some basic constructions of the geometric drawing, as angles and agreements, had been rescued for the drawing course.

The techniques of constructions using rulers had been adapted for a sketch supported by printed grids. The student now produces a sketch more elaborated, with geometric properties. The trainings daily develop the manual ability and the geometric reasoning.

This paper shows the techniques used for the confection of sketches to the hand-free one with geometric properties, as presented to follow.

### 3 Techniques used

During the first semester, the approach of discipline is the orthographics representations, or either, two dimensions are only represented in each projection. For the last semester, as much the orthographics representations as the isometric axonometric perspective representations are boarded. Although the students have that to develop graphical representations using two or three dimensions, the presented techniques can be applied in the two types of representation.

As previously presented, for the ability of the hand-free sketch development, the students have a printed matter instrument support in the proper leaf of the exercises. From a square-lined (in the bidimensional representation) or isometric(in the three-dimensional representation) griding, the students can use a set of references, as well as horizontal and vertical directions on orthographics representation, or the directions, unbalanced of  $120^\circ$  for the isometrics perspectives representation. Moreover, the set of the lines intersections that compose the grid generates a set of points equally spaced, establishing a unit of measure and each one of two or three represented directions.

The Figures 1 and 2 shows the grids printed in the students exercises' leaf.



Figure 1: Orthographics grid.

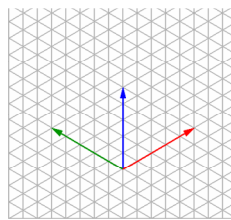


Figure 2: Isometric perspective grid.

The students developed sketch must contain the projected objects characteristics (considering faces and edges visible and obstructed by other elements), beyond its properties, as ratio, positioning and inclination. They are, therefore, requested to do not erase the used geometric constructions. This allows teachers verify the visualization and construction developed process for the students during the resolution of the exercise.

In this way, the lines used for the geometric construction must visibly be differentiated of the too much traces of the drawing. The thickness of these lines is lesser of that the elements that really belong to the object.

#### 3.1 Basic geometric constructions

This section will be presented the basic techniques for the construction of sketches using the grid as support instrument. The examples will be presented for representations of only two dimensions, but they can be extended for three, considering that the grid for three-dimensional representations is formed by the overlapping of three bidimensional grids, orthogonal ones between itself, two by two.

#### Parallels strait line bars

Once presented the straight line segments drawing technique, it's become necessary that these are, in its majority, parallels or perpendiculars between itself. The construction of a parallel straight line to another one gives from the analysis of the equation of two parallel straight lines bars, represented for Equations 1 and 2.

$$r: y = a \cdot x + b_1 \quad (1)$$

$$s: y = a \cdot x + b_2 \quad (2)$$

Notice that both the equations possess the same slope **a**. To leave of this particularity, the technique is presented to the students to construct a parallel segment of straight line to another one.

So, a rectangular triangle must be defined, using part of segment **r** as hypotenuse. The cathetus are defined by the horizontal and vertical directions. Once the grid represents the system of coordinated directions and also defines the unit in each direction, the process for the triangle construction is facilitated in the situations where the limit hypotenuse points and the intersections of the grid coincide.

The triangle geometric definition results in cathetus with dimensions **H** and **V**, measured in the grid unit defined. From a known point, a line segment **s**, **r** parallel, can be traced from a right triangle, of cathetus **H** and **V**, as it can be observed in the Figure 3:

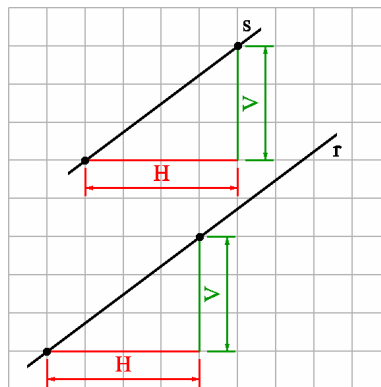


Figure 3: Parallel strait lines.

#### Perpendicular straight lines

In analogous way, the equations of two perpendicular straight lines between itself can be analyzed with intention to verify as a graphical construction can be elaborated.

$$r: y = a \cdot x + b \quad (3)$$

$$s: y = -\frac{1}{a} \cdot x + c \quad (4)$$

Using the same technique showed for the construction of parallel straight lines bars, a stretch of straight line **r** can be defined a triangle, of cathetus **H** and **V**, considering as hypotenuse. Of this form, the absolute value of the slope of straight lines **r** and **s** can be express in the following way:

$$a = \frac{V}{H} \Leftrightarrow \frac{1}{a} = \frac{H}{V} \quad (5)$$

By the Equation 5, it can be noticed that to draw a straight line **s**, perpendicular to **r**, must be changed the value of the horizontal direction with the one of the vertical line. Although the characteristics of the triangle are defined, Equation 4 indicates that the slope is negative. This result in the construction of one of its cathetus (**V** or **H**) opposing to the direction defined for the system of coordinates presented in Figure 1.

The figure 4 exemplifies the process:

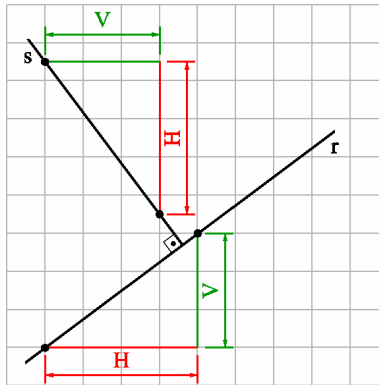


Figure 4: Perpendicular strait lines.

Another didactic approach consists on the use of geometric transformations to make the rotation, of an  $90^\circ$  angle, between the straight line segments. In this matter, the triangle formed by the cathetus **H** and **V** must be considered, whose hypotenuse belongs to segment **r**.

Placed the rotation transformation, for Equation 6, can be verified that the inversion of the values is equivalent to the presented one in the first approach, including the direction that the values must be drawn, following the system of coordinates used.

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} \cos(90^\circ) & -\sin(90^\circ) \\ \sin(90^\circ) & \cos(90^\circ) \end{bmatrix} \cdot \begin{bmatrix} H \\ V \end{bmatrix} \\ \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} H \\ V \end{bmatrix} = \begin{bmatrix} -V \\ H \end{bmatrix} \end{aligned} \quad (6)$$

This approach, even so of simple resolution, is not applied under the mathematical form, a time that the concepts of geometric transformations had not been presented to the students

until then. However, visually this approach can be understood more easily with the use of multimedia resources, as, for example, an animation that represents the rotation of one of the triangles while the other remains static. In this way, it is easy to perceive that values **H** and **V** change of position during the creation of the new triangle.

#### Circumferences

Another basic construction that is used as prerequisite for the creation of angles and tangency is the circumference or a circumference arc.

Another basic construction that is used as prerequisite for the creation of angles and agreement is the circumference or a circumference arc. The construction of the circumferences will be based on the property of the perpendicularity between a tangent straight line the circumference and a segment line that joins the center **O** to the tangency point **T** [3], as figure 5 shows.

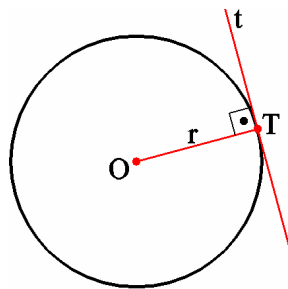


Figure 5: Tangent straight line to the circumference.

This property will be adjusted to the use of the grid, tracing two horizontal straight lines and two vertical straight lines tangent to the circumference. In the practical one, the construction of the circumference is initiated by the construction of a square whose sides represent its diameter and its center must coincide with the center of the circumference, as shown in the figure 6:

From the middle points of the edges that compose the square, small arcs must be tracings. These arcs must be drawn out, little by little, until the diagonal lines to the square meet next, as shown in the Figure 7:

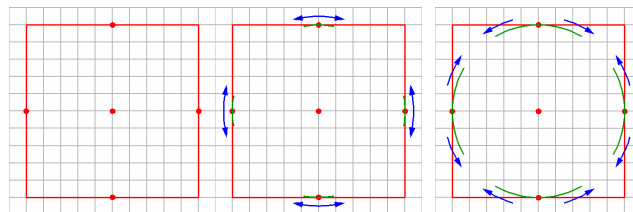


Figure 6: Tangent straight line to the circumference and construction of the circumference.

The arcs approach process must be evaluated constantly, erasing the stretches that, visually, do not correspond to a circumference. With the practical issues, the students can develop this ability, expending little time when drawing the circumferences.

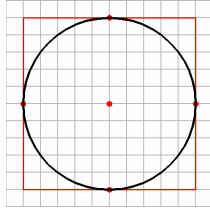


Figure 7: Finished circumference and its lines of construction.

To trace only one  $90^\circ$  opening arc, a square must be created whose side represents the ray of the circumference.

### 3.2 The properties application

From the construction of straight lines parallel bars, perpendiculars and circumferences arcs, the students can make combinations to draw angles and elements of agreement. This section shows some examples of these properties application.

#### Angles

In some situations, the relations between the elements of an object are express under the angular form. From two arcs of circumference, angles with  $15^\circ$  precision can be traced. For such, a square of convenient size must be traced. The centers of the circumferences arcs must be coincident with the square vertices. Moreover, the arc ray must have the same value of the square side. The figure 8 shows the four possible arcs that can be drawn in the interior of the square.

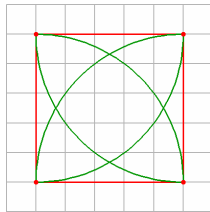


Figure 8: Construction of angles.

The intersection between the arcs circumference generates multiple of  $15^\circ$  angles. Mathematically, the validity of the diagram used, represented at the figure 8, can be verified deciding, two the two, the equations to follow:

$$\begin{aligned}
 (I) \quad & x^2 + y^2 = r^2 \\
 (II) \quad & (x-r)^2 + y^2 = r^2 \\
 (III) \quad & x^2 + (y-r)^2 = r^2 \\
 (IV) \quad & (x-r)^2 + (y-r)^2 = r^2
 \end{aligned} \tag{7}$$

Table 1 shows that the solution of the pairs of equations results in the desired angles:

Tabela 1: Angles.



Equations	Result angle
$(III) \cap (IV)$	$15^\circ$
$(I) \cap (III)$	$30^\circ$
$(I) \cap (II)$	$60^\circ$
$(II) \cap (IV)$	$75^\circ$

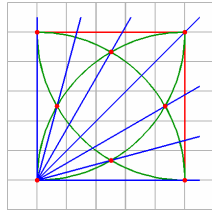


Figure 9: Multiple  $15^\circ$  angles.

### Tangency

Beyond the constructions of the angles, some objects possess cylindrical tangency of plans or surfaces between the elements that compose them. Considering the situation of the orthographic representation, the tangency gives for the soft transition between two elements that finds it in a unique point, called tangency point [3, 4].

The follow example shows how properties and presented techniques can be used to trace an circumference arc of 10 ray tangent to others two, of known center and ray.

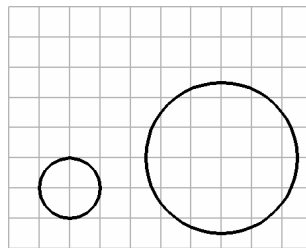


Figure 10: Known circumferences.

To trace the 10 ray circumference, its center must be determined. This point can be gotten by the locus intersection of the points that equidistant the known circumferences 10 units each.

Another circumference must be traced in same center of the original ones, with ray plus the 10 ray of the circumference, as shows to the figure 11:

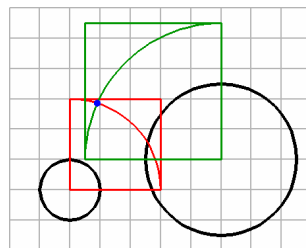


Figure 11: The center localization.

It can be observed that for the circumferences tracing equal sides squared had been used to the circumferences rays. The intersection point indicates the desired circumference center position. To leave of this point, a new circumference will be drawn, using the presented techniques.

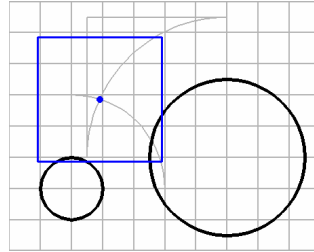


Figure 12: Circumference delimitation.

Figure 13 shows, in prominence, the solution of the problem:

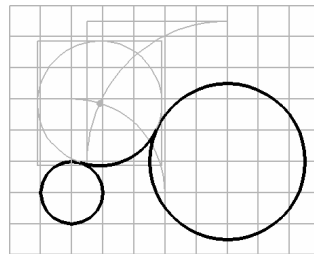


Figure 13: Final solution.

This sequence showed only one example of the technique application results, however tangency can be made between circumferences and two straight lines; straight line segments tangency two circumferences, beyond the combinations between these elements.

### 3.3 Technical drawing applications

The Technical Drawing course approach of the Escola de Engenharia Mauá is centered in the reading, interpretation and confection of a object graphical representations under the form of a set of orthographics or a isometric perspective representation. Or either, the until presented examples are mere didactic elements used for the concepts communication that will be applied later. The Figure 14 shows to an example of the techniques application for the three orthographics sketches representation of an object, while the Figure 15 shows the same object construction, represented at isometric perspective. For convention, it was used the first dihedron projection.

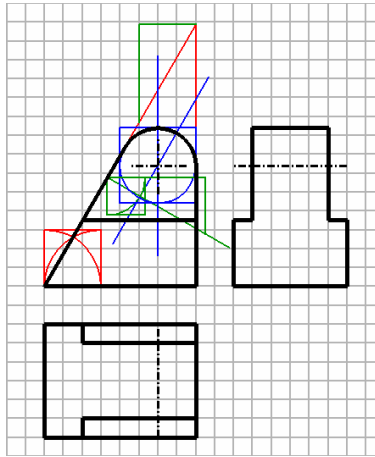


Figure 14: Geometric constructions in orthographics.

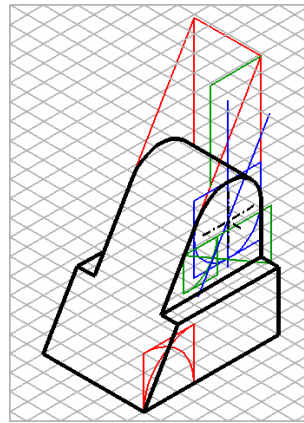


Figure 15: Geometric constructions in isometric perspective.

## 4 Conclusions

The use of geometric drawing in the sketch makes the student to think geometry while it draws, creating an essential instinct to the project designer in formation.

Applied geometry is the base of the interaction between the engineer course disciplines. The acquired knowledge strengthens the bases for disciplines as Physics, Mechanics and Resistance of the Materials.

Among the years, the research team observed the students using the hand sketches in many situations as resource to solve their engineer's problems.

## Gratefulness

To all the professors of Drawing discipline who participate in active way for the development and perfection of didactic methodologies and the Escola de Engenharia Mauá for allowing that didactic actions as these can be developed and applied.

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